

① DEFINITION & NOTATION

If $b^2 - 4ac < 0 \rightarrow$ complex roots

Form: $z = a + bi = (a, b)$

• $a = \text{Re}(z) = \text{real part}$

• $b = \text{Im}(z) = \text{imaginary part}$

$i = \sqrt{-1} \quad i^2 = -1$

$i^3 = -i \quad i^4 = 1 \quad (\text{cycle of } 4)$

★ $(-1)^{\text{even}} = 1 \quad ; \quad (-1)^{\text{odd}} = -1$

Tip: find $i^n \rightarrow$ divide n by 4, use remainder

② +, -, ×, ÷

Add / Subtract: real to real, imag to imag

Multiply: like algebra, replace i^2 with -1

Conjugate of $z = a + bi$:

$\bar{z} = a - bi$ (flip sign of imag part)

$z \cdot \bar{z} = \text{real number } (a^2 + b^2)$

Divide: $\times$ top & bottom by conjugate of bottom

Equality: if $z_1 = z_2$ then real = real, imag = imag

③ SQUARE ROOTS OF COMPLEX NO.

4-Step Method:

1. Let $\sqrt{\text{given}} = a + bi$
2. Square both sides
3. Real = Real, Imag = Imag
4. Solve simultaneous equations for a, b

④ ARGAND DIAGRAM & MODULUS

x-axis = Real axis

y-axis = Imaginary axis

Modulus: $|z| = \sqrt{a^2 + b^2}$ — distance from origin

Transformations of z :

- $kz \rightarrow$ dilation ($k > 1$ out, $k < 1$ in, $k < 0$ opposite)
- $iz \rightarrow 90^\circ$ rotation anticlockwise
- $z + w \rightarrow$ forms a parallelogram
- $\bar{z} \rightarrow$ directly below (or above) z

⑦ POLAR FORM

$z = r(\cos \theta + i \sin \theta)$

• $r = |z| = \sqrt{x^2 + y^2}$

• $\tan \theta = y/x$ ($\theta = \text{argument}$)

General polar form:

$z = r(\cos(\theta + 2n\pi) + i \sin(\theta + 2n\pi))$

! **WATCH:** Once a fraction is in the power, **MUST** use general polar form

Properties:

$z_1 z_2 = r_1 r_2 (\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2))$

$z_1 / z_2 = (r_1 / r_2) (\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2))$

$1/z_1 = (1/r_1) (\cos \theta_1 - i \sin \theta_1)$

⑤ QUADRATIC EQUATIONS

Type 1: Find roots $\rightarrow$ formula $x = [-b \pm \sqrt{b^2 - 4ac}] / 2a$

(works whether a, b, c are real or unreal)

Type 2: Form quadratic from roots:

$z^2 - (\alpha + \beta)z + \alpha\beta = 0$

Type 3: Find 2nd root given 1st: same formula

Type 4: Find unknown coefficient: sub root in

★ If coefficients are **REAL**, complex roots come in **CONJUGATE** pairs

⑥ CUBIC EQUATIONS

If z_1 is a complex root and coefficients real:

$\rightarrow \bar{z}_1$ is also a root

To find 3rd root:

1. Form quadratic from z_1 and $\bar{z}_1$
2. Divide cubic by that quadratic
3. The remaining linear factor gives 3rd root

Form cubic from 3 roots:

$(z - r_1)(z - r_2)(z - r_3) = 0$

⑧ DE MOIVRE'S THEOREM

If $z = r(\cos \theta + i \sin \theta)$, then:

$z^n = r^n (\cos n\theta + i \sin n\theta)$

★ Power of z multiplies the angle, raises the modulus

Use it for:

- Raising to a power: e.g. $(2-2i)^4$
- Finding roots: e.g. $z^3 = 1 \rightarrow$ cube roots
- Proving trig identities ($\cos 4\theta, \sin 3\theta$, etc.)

Roots on a diagram:

- $r > 1 \rightarrow$ spiral OUT
- $r = 1 \rightarrow$ unit circle
- $r < 1 \rightarrow$ spiral IN

⑨ TRIG PROOFS via DE MOIVRE

Standard expansion to know:

$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$

To prove e.g. $\sin 4\theta = 4 \sin \theta (2\cos^3 \theta - \cos \theta)$:

1. Expand $(\cos \theta + i \sin \theta)^4$ binomially
2. Equate REAL parts $\rightarrow \cos 4\theta$ identity
3. Equate IMAG parts $\rightarrow \sin 4\theta$ identity

For $\tan n\theta$: $\tan n\theta = \sin n\theta / \cos n\theta$

⑩ TRIG IDENTITY (POWERS OF cos/sin)

Prove $\cos^3 \theta = \frac{1}{4}(\cos 3\theta + 3 \cos \theta)$:

Use: $z + 1/z = 2 \cos \theta \quad ; \quad z^n + 1/z^n = 2 \cos n\theta$

$z - 1/z = 2i \sin \theta$

Method: Cube $(z + 1/z)^3 = (2 \cos \theta)^3$, expand LHS, group $z^n + 1/z^n$ pairs, replace with $2 \cos n\theta$, rearrange.