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DIFFERENTIATION

Exam Cog

① DIFFERENTIATION FROM 1ST PRINCIPLES

5-Step method:
1. Let f(x) = (function given)
2. To every x, add h: f(x+h)
3. Subtract: f(x+h) − f(x)
4. Divide across by h
5. Let h → 0 → anything with h goes to 0

✓ **TIP:** *Follow LHS down exactly — work is in algebra/trig on RHS*

② BASIC RULE

If $y = x^n$ then $dy/dx = n \cdot x^{n-1}$

★ *Multiply by the power, REDUCE the power by one*

Say it over and over — you'll mix it up with integration!
Don't forget: rules of indices, use the tables

③ PRODUCT, QUOTIENT, CHAIN

Product Rule (u × v):
(uv)' = u'v + uv'

★ *Leave, Diff + Diff, Leave*

Quotient Rule (u ÷ v) — in tables:
$(u/v)' = (u'v - uv') / v^2$
Tip: try algebra/factor cancel first!
Chain Rule (function of a function):
dy/dx = (du/dx) × (dy/du)
Quick: differentiate INSIDE × differentiate OUTSIDE

④ e^x, ln, $\sin^3 x$, IMPLICIT

Exponential: d/dx(e^f(x)) = f'(x) · e^f(x)
→ Diff the power × leave e the same
Log: d/dx(ln f(x)) = f'(x) / f(x)
→ Diff × (1 over the function)
Note: ln e = 1 ; $\sin^3 x = (\sin x)^3$ → chain
Implicit: $d/dx(y^2) = 2y \cdot (dy/dx)$

⚠ **WATCH:** $d^2x/dy^2 \neq (dy/dx)^2$

⑤ SLOPE OF A TANGENT

Tangent line needs: a point + a slope
m = dy/dx at the point (x, y)
Other slope formulae:
• $m = (y_2 - y_1)/(x_2 - x_1)$ • m = rise/run
• Line ax+by+c=0 → m = −a/b
• m = tan A (angle with positive x-axis)

★ *At point of contact: slope of curve = slope of tangent*

Got x? Sub back to original to get y

⑥ INC/DEC & MAX/MIN POINTS

Increasing: dy/dx > 0 **Decreasing:** dy/dx < 0
Stationary / turning: dy/dx = 0
Max: dy/dx = 0 AND $d^2y/dx^2 < 0$
Min: dy/dx = 0 AND $d^2y/dx^2 > 0$
Inflection: $d^2y/dx^2 = 0$
Max/Min word problems — 5 steps:
1. What's max/min of? 2. Formula: A = ...
3. Equation linking variables
4. Get one variable in terms of other
5. Sub into step 2, diff, set = 0

⚠ **WATCH:** UNITS are vital — always state them!

⑦ RATES OF CHANGE

Type 1: Distance / Speed / Acceleration
• s = f(t) = distance • v = ds/dt = speed
• $a = d^2s/dt^2$ = acceleration
• Initial / at rest: t = 0 • Max speed: ds/dt = 0
Type 2: Linked rates (chain across rates)
dV/dt = (dV/dr) × (dr/dt)
dr/dt → m/s dA/dt → m^2/s dV/dt → m^3/s

★ *What do we have? What do we need? LINK is in the question.*

⚠ **WATCH:** DON'T FORGET UNITS