

① NUMBER SETS

- N** Naturals (>0)
- Z** Integers (+/- whole)
- Q** Rationals (fractions)
- R** Reals (any number)
- R-Q** Irrationals

② NOTATION & PERFECT SQUARES

In $3x^2 + 7x + 9$:

- **3** = coefficient
- **x** = variable
- **2** = power / degree
- **9** = constant

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

★ Square 1st, square 2nd, 1st × 2nd doubled

③ FACTORISING — 5 TYPES

- Grouping** (2 or 4 terms): take out common
- Quadratic trinomial**: guide-no. / double brackets / big X
- Diff of two squares**:

$$a^2 - b^2 = (a-b)(a+b)$$
- Sum/diff of cubes**:

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$
- Combinations**: take out common first

⑤ FRACTIONS & SIMULTANEOUS EQNS

Fractions: Add/Sub/Equality → common denominator. Complex: simplify top & bottom to single fractions, then flip & multiply.

Simultaneous Equations:

- Two lines (basic)
- Line + curve: from linear get x= or y=, sub into curve
- Substitution e.g. let $a=1/x$, $b=1/y$
- 3×3 system — multiply WHOLE line, watch signs
- Identity: equate coeffs of x & constants both sides

⑧ SURDS

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \quad \sqrt{a/b} = \sqrt{a}/\sqrt{b}$$

$$(\sqrt{a})^2 = a \quad \sqrt{a} \cdot \sqrt{a} = a$$

⚠ **WATCH**: $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$

Simplify: factor out perfect square

Add/sub: only same irrational part

Conjugate: flip middle sign ($a+\sqrt{b} \leftrightarrow a-\sqrt{b}$). Product = rational.

Rationalise: single surd → ×surd; compound → ×conjugate

Surd eqns: isolate $\sqrt{\quad}$, square both sides, solve, **CHECK** ($\sqrt{\quad}$ is + only)

④ QUADRATIC EQN $ax^2+bx+c=0$

- Factorise & let factors = 0, OR
- $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Discriminant $b^2 - 4ac$:

- > 0 → 2 real roots
- < 0 → complex (unreal)
- = 0 → equal roots (perfect sq)
- ≥ 0 → real roots

★ Complete the square: $\frac{1}{2}(\text{coeff of } x)$, then square

⑥ ROOTS & FACTOR THEOREM

Roots of quadratic (coeff of $x^2 = 1$):

$$x^2 - (\text{sum of roots})x + (\text{product}) = 0$$

Factor Theorem: if $f(k)=0$ then $(x-k)$ is a factor

Methods: sub in OR divide in

Types: find factors (try $\pm 1, \pm 2, \dots$); 1 unknown (sub root, =0); 2 unknowns; variable factor (mult quadratic × linear $(x+k)$)

⑦ INEQUALITIES & $|x|$

$|x|$: always ≥ 0. Methods: (1) split \pm cases (2) square both sides (3) graph it

Inequalities: treat like equations BUT flip sign when × or ÷ by negative

Quadratic: solve =0; smaller root, x, larger root with sign — sketch graph

With fractions: × by denominator SQUARED (always +, sign safe)

With $| \cdot |$: square both sides

Proofs: perfect squares backwards, or use given info ($a>b \Rightarrow a-b>0$)

⑨ INDICES

Rules in tables. To solve: **get same base**, then equate powers

⚠ **WATCH**: $2(2^x) \neq 4^x$ — it's 2^{x+1}

Quadratic in indices: e.g. $3^{2x+1} = 3 \cdot (3^x)^2 \rightarrow$ let $t = 3^x$

Check answers: positive base \neq negative

⚠ **WATCH**: $5 \cdot 3^n \neq 15^n$ (very common slip!)

⑩ LOGS

Definition: $\log_b(n) = p \Leftrightarrow b^p = n$

★ **BNP** — Base, to power of P, equals Number

Rules: $\log(ab) = \log a + \log b$

$$\log(a/b) = \log a - \log b$$

$$\log a^n = n \cdot \log a$$

⚠ **WATCH**: $(\log a)^n \neq n \cdot \log a$; $\log a / \log b \neq \log(a-b)$

Type 1: log = number — combine logs, use BNP

Type 2: logs = logs — drop logs

Type 3: Change of base — break the bigger one down

Type 4: x in power — log both sides

Always check: log of negative not allowed