

① THE FOUR STEPS

Every area or volume question

- 1. Shape & information — what is it, what are you told?
- 2. Formula — write the right one down first.
- 3. Put the figures in and work it out.
- 4. Units are vital — always write them.

★ *Perpendicular height = straight down from the tip to the base, not the slant. It is the H in $\frac{1}{2}BH$.*

② AREA & PERIMETER

Rectangle

$$A = \ell \times b \quad P = 2(\ell + b)$$

Triangle — three ways

$$A = \frac{1}{2} BH \quad (\text{base} \times \text{perp. height})$$

$$A = \frac{1}{2} ab \sin C \quad (\text{two sides} + \text{angle})$$

$$A = \frac{1}{2} |x_1y_2 - x_2y_1| \quad (\text{corner at origin})$$

Parallelogram

$$A = a h \quad \text{or} \quad A = a b \sin C \quad (\text{no half})$$

Circle

$$A = \pi r^2 \quad \text{Circumference } \ell = 2\pi r$$

★ *Sine gives two angles, C and $180^\circ - C$ — for a parallelogram either can be correct.*

③ USING π

$\pi = 3.14$ or $\pi = \frac{22}{7}$ — use what the question asks.

★ *"In terms of π " → leave the π in, do not multiply it out.*

④ SECTORS

In degrees

$$\text{Area } A = \pi r^2 (\theta/360)$$

$$\text{Arc length} = 2\pi r (\theta/360)$$

In radians

$$\text{Area } A = \frac{1}{2} r^2 \theta \quad \text{Arc } \ell = r\theta$$

• θ must be in radians · $180^\circ = \pi$ radians.

Perimeter of a sector

$$P = 2r + \ell \quad (\text{two radii} + \text{the arc})$$

• A reflex sector uses the same formulas — the angle is just bigger than 180° .

⑤ COMPOUND, SHADED & MAX/MIN

Compound & shaded shapes

- Joined together → add the areas.
- One cut out of another → Outside — Inside.

Split into rectangles, triangles or circles; find each, then add or subtract.

★ *Different units? Convert so they match before you start.*

Maximum & minimum area

- 1. Write the area as a formula.
- 2. Reduce to one variable using the given info.
- 3. Differentiate and let $dA/dx = 0$.
- 4. Solve, then find the area.

⑥ SOLIDS — BOX & CYLINDER

Surface area = the total area of all the faces of a solid.

Cube & cuboid

$$V = \ell \times b \times h$$

$$A = 2(\ell b + \ell h + bh)$$

Cylinder

$$V = \pi r^2 h \quad \text{Curved } A = 2\pi r h$$

$$\text{Each end circle} = \pi r^2$$

★ *Open top = no lid. Open cylinder = curved + 1 circle; solid = curved + 2 circles.*

⑦ CONE, SPHERE & HEMISPHERE

Cone

$$\text{Slant } \ell^2 = h^2 + r^2$$

$$V = \frac{1}{3} \pi r^2 h \quad \text{Curved } A = \pi r \ell$$

★ *Curved area needs the slant ℓ , not the vertical h — find ℓ first.*

Sphere

$$V = \frac{4}{3} \pi r^3 \quad A = 4\pi r^2$$

Hemisphere

$$V = \frac{2}{3} \pi r^3 \quad \text{Curved } A = 2\pi r^2$$

$$\text{Solid total } A = 2\pi r^2 + \pi r^2 = 3\pi r^2$$

✓ *Working backwards: put the known value in, keep the unknown on the left, solve.*

⑧ NETS

A net is a solid opened out flat.

- Box → six rectangles.
- Cylinder → two circles + one rectangle (height = h , width = circumference $2\pi r$).
- Cone → one circle (base) + one sector (straight edge = slant ℓ , arc = $2\pi r$).

⑨ COMBINED SOLIDS & RECASTING

Double shapes

- Volume → add the two volumes.
- Surface area → add only the faces you can see; drop any hidden where they meet.

Meltdown & recast

- Volume stays the same: set $V_{\text{old}} = V_{\text{new}}$ and solve.
- "How many can be made?" → divide, then round DOWN to a whole shape.

"Just fits"

- Ball in a box → box side = diameter.
- Ball in a cylinder → height = diameter, radius = the ball's radius.

★ *Carl's trick: a front-end (2-D) sketch shows the dimensions at a glance.*

⑩ EMPTY SPACE, FLOW & UNITS

Empty space & water

Empty space = container — solid

$$\% = (\text{space}/\text{container}) \times 100$$

• Water displacement: the risen band is a cylinder — container radius, volume = the object.

Rate of flow

$$\text{Volume per second} = \pi r^2 \times \text{speed}$$

$$\text{Time} = \text{total volume}/(\text{volume per second})$$

Units & conversions

$$\text{length } m \cdot \text{area } m^2 \cdot \text{volume } m^3$$

$$\bullet 1 \text{ litre} = 1000 \text{ cm}^3 \cdot 1 \text{ m} = 100 \text{ cm}$$

$$\bullet 1 \text{ m}^2 = 10\,000 \text{ cm}^2 \cdot 1 \text{ m}^3 = 1\,000\,000 \text{ cm}^3$$

★ *Match units before you start (e.g. $80 \text{ cm} = 0.8 \text{ m}$).*