# THE CIRCLE

**Exam Cog — Need to Know**

## ① NOTATION

**Centre:** exact mid-point of circle

**Radius:** centre to any point on circle

**Diameter:** line through centre across circle

**Chord:** line hitting circle at 2 points (diameter is special chord)

**Tangent:** touches circle at ONE point only

★ *The two main parts: CENTRE and RADIUS*

## ② EQUATIONS OF A CIRCLE

**Centre (0, 0), radius r:**

$x^2 + y^2 = r^2$

**Centre (h, k), radius r:**

$(x - h)^2 + (y - k)^2 = r^2$

**General form:** $x^2 + y^2 + 2gx + 2fy + c = 0$

• Centre = $(-g, -f)$

• Radius = $\sqrt{(g^2 + f^2 - c)}$

✓ **TIP:** ALWAYS write down centre & radius first — attempt marks!

## ③ POINT IN, ON or OUTSIDE?

**Sub $(x_1, y_1)$ into LHS of circle equation, compare to $r^2$:**

• LHS $<$ $r^2$ → point is **INSIDE**

• LHS $=$ $r^2$ → point is **ON the circle**

• LHS $>$ $r^2$ → point is **OUTSIDE**

## ④ LINE MEETS CIRCLE

**Solve simultaneously** (line into circle):

• 2 real answers → line is a **CHORD**

• 1 real answer → line is a **TANGENT**

• 2 complex answers → line **DOES NOT touch**

**Better way to PROVE tangent (no contact pt asked):**

Perpendicular distance from centre = radius

**3 cases via perp distance:**

• d = r → tangent • d < r → chord

• d > r → doesn't touch

## ⑤ TANGENT AT POINT OF CONTACT

**4-Step method:**

1. Find centre
2. Slope from centre to contact point
3. INVERT and CHANGE SIGN → slope of tangent
4. Use point + slope: $y - y_1 = m(x - x_1)$

★ *Tangent ⊥ radius at point of contact*

**Quick way** (centre = (0,0) only):

Tangent at $(x_1, y_1)$ on $x^2 + y^2 = r^2$ is:

$x_1x + y_1y = r^2$

## ⑥ WHERE CIRCLE CUTS AXES

**x-axis:** let y = 0, solve for x

**y-axis:** let x = 0, solve for y

**Circle TOUCHING the axes:**

• Touches x-axis → $g^2 = c$

• Touches y-axis → $f^2 = c$

**Image under transformation:**

Radius STAYS the same — only transform the centre

## ⑦ FINDING THE EQUATION

**From end-points of a diameter:**

• Centre = midpoint of the two points

• Radius = distance from centre to either

★ *Right-angled triangle? Centre = midpoint of HYPOTENUSE*

**Through 3 given points:**

Sub each into $x^2+y^2+2gx+2fy+c=0$ → 3 eqns in g, f, c

**Through 2 points, centre on a given line:**

• Sub the 2 points into general form

• Sub centre $(-g, -f)$ into the line equation

→ 3 equations, solve for g, f, c

## ⑧ KEY PROPERTY & EXTERNAL TANGENTS

★ *Radius ⊥ to chord → BISECTS the chord*

**Tangents from a point OUTSIDE a circle:**

1. Let line be $y - y_1 = m(x - x_1)$
2. Sub in the given outside point
3. Perpendicular distance from centre = radius
4. Solve for m (usually two values → two tangents)

## ⑨ COMMON CHORDS & TANGENTS

**Two circles:** $S = 0$ and $S_1 = 0$

**$S - S_1 = 0$** gives a STRAIGHT LINE which is:

• **Common chord** if S and $S_1$ meet at 2 points

• **Common tangent** if they touch (1 point only)

✓ **TIP:** Subtract the two circle equations — the $x^2$ and $y^2$ cancel!

## ⑩ TOUCHING CIRCLES

**Two circles, radii $r_1$ and $r_2$, distance d between centres:**

• **External touch:** $d = r_1 + r_2$

(small circle outside the other, touching)

• **Internal touch:** $d = |r_1 - r_2|$

(one circle inside the other, touching)

★ *Add radii for outside, subtract radii for inside*