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COMPLEX NUMBERS
Exam Cog

① DEFINITION & NOTATION

If $b^2 - 4ac < 0$ → complex roots

Form: $z = a + bi = (a, b)$

- **a** = Re(z) = real part
- **b** = Im(z) = imaginary part

$i = \sqrt{-1}$ $i^2 = -1$

$i^3 = -i$ $i^4 = 1$ (cycle of 4)

★ *$(-1)^{even} = 1$; $(-1)^{odd} = -1$*

Tip: find i^n → divide n by 4, use remainder

② +, −, ×, ÷

Add / Subtract: real to real, imag to imag

Multiply: like algebra, replace i^2 with −1

Conjugate of $z = a + bi$:

$\bar{z} = a - bi$ (flip sign of imag part)

$z \cdot \bar{z}$ = real number $(a^2 + b^2)$

Divide: × top & bottom by conjugate of bottom

Equality: if $z_1 = z_2$ then real = real, imag = imag

③ SQUARE ROOTS OF COMPLEX NO.

4-Step Method:

1. Let $\sqrt{\text{given}} = a + bi$
2. Square both sides
3. Real = Real, Imag = Imag
4. Solve simultaneous equations for a, b

④ ARGAND DIAGRAM & MODULUS

x-axis = Real axis

y-axis = Imaginary axis

Modulus: $|z| = \sqrt{a^2 + b^2}$ — distance from origin

Transformations of z:

- **kz** → dilation (k>1 out, k<1 in, k<0 opposite)
- **iz** → 90° rotation anticlockwise
- **z + w** → forms a parallelogram
- **$\bar{z}$** → directly below (or above) z

⑦ POLAR FORM

$z = r(\cos\theta + i\sin\theta)$

- $r = |z| = \sqrt{x^2 + y^2}$
- $\tan\theta = y/x$ (θ = argument)

General polar form:

$z = r(\cos(\theta + 2n\pi) + i\sin(\theta + 2n\pi))$

⚠️ **WATCH:** Once a fraction is in the power, MUST use general polar form

Properties:

$z_1z_2 = r_1r_2(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$

$z_1/z_2 = (r_1/r_2)(\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2))$

$1/z_1 = (1/r_1)(\cos\theta_1 - i\sin\theta_1)$

⑤ QUADRATIC EQUATIONS

Type 1: Find roots → formula $x = [-b \pm \sqrt{b^2-4ac}] / 2a$

(works whether a, b, c are real or unreal)

Type 2: Form quadratic from roots:

$z^2 - (\alpha+\beta)z + \alpha\beta = 0$

Type 3: Find 2nd root given 1st: same formula

Type 4: Find unknown coefficient: sub root in

★ *If coefficients are REAL, complex roots come in CONJUGATE pairs*

⑥ CUBIC EQUATIONS

If z_1 is a complex root and coefficients real:

→ $\bar{z}_1$ is also a root

To find 3rd root:

1. Form quadratic from z_1 and $\bar{z}_1$
2. Divide cubic by that quadratic
3. The remaining linear factor gives 3rd root

Form cubic from 3 roots:

$(z - r_1)(z - r_2)(z - r_3) = 0$

⑧ DE MOIVRE'S THEOREM

If $z = r(\cos\theta + i\sin\theta)$, then:

$z^n = r^n(\cos n\theta + i\sin n\theta)$

★ *Power of z multiplies the angle, raises the modulus*

Use it for:

- Raising to a power: e.g. $(2-2i)^4$
- Finding roots: e.g. $z^3 = 1$ → cube roots
- Proving trig identities (cos 4θ, sin 3θ, etc.)

Roots on a diagram:

- r > 1 → spiral OUT • r = 1 → unit circle
- r < 1 → spiral IN

⑨ TRIG PROOFS via DE MOIVRE

Standard expansion to know:

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

To prove e.g. $\sin 4\theta = 4\sin\theta(2\cos^3\theta - \cos\theta)$:

1. Expand $(\cos\theta + i\sin\theta)^4$ binomially
2. Equate REAL parts → cos 4θ identity
3. Equate IMAG parts → sin 4θ identity

For tan nθ: $\tan n\theta = \sin n\theta / \cos n\theta$

⑩ TRIG IDENTITY (POWERS OF cos/sin)

Prove $\cos^3\theta = \frac{1}{4}(\cos 3\theta + 3\cos\theta)$:

Use: $z + 1/z = 2\cos\theta$; $z^n + 1/z^n = 2\cos n\theta$

$z - 1/z = 2i\sin\theta$

Method: Cube $(z + 1/z)^3 = (2\cos\theta)^3$, expand LHS, group $z^n + 1/z^n$ pairs, replace with $2\cos n\theta$, rearrange.