

Differentiation

Exam Cog

1 Differentiation from 1st principles

5-Step method:

1. Let $f(x)$ = (function given)
2. To every x , add h : $f(x+h)$
3. Subtract: $f(x+h) - f(x)$
4. Divide across by h
5. Let $h \rightarrow 0 \rightarrow$ anything with h goes to 0

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

★ Follow LHS down exactly — work is in algebra/trig on RHS

2 Basic rule

$$\text{If } y = x^n \text{ then } \frac{dy}{dx} = n \cdot x^{n-1}$$

★ Multiply by the power, REDUCE the power by one

Say it over and over — you'll mix it up with integration!

Don't forget: rules of indices, use the tables

Get into standard form first:

- Multiply out brackets
- Negative powers: $1/a^p = a^{-p}$
- Roots: $\sqrt{x} = x^{1/2}$
- Fractions: factorise or split top over bottom

3 Standard derivatives — from the tables

$$y = \sin x \rightarrow \frac{dy}{dx} = \cos x$$

$$y = \cos x \rightarrow \frac{dy}{dx} = -\sin x$$

$$y = \ln x \rightarrow \frac{dy}{dx} = \frac{1}{x}$$

$$y = e^x \rightarrow \frac{dy}{dx} = e^x$$

4 Product, Quotient, Chain

Which rule? By \rightarrow Product \cdot Divide \rightarrow Quotient \cdot Of \rightarrow Chain

Product Rule ($u \times v$):

$$(uv)' = u'v + uv'$$

★ Leave, Diff + Diff, Leave

Quotient Rule ($u \div v$) — in tables:

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

Tip: try algebra/factor cancel first!

Chain Rule (function of a function):

$$\frac{dy}{dx} = \left(\frac{du}{dx}\right) \times \left(\frac{dy}{du}\right)$$

Quick: differentiate INSIDE \times differentiate OUTSIDE

5 e^x , \ln , $\sin 3x$, implicit

Exponential: $\frac{d}{dx}(e^{f(x)}) = f'(x) \cdot e^{f(x)}$

\rightarrow Diff the power \times leave e the same

Log: $\frac{d}{dx}(\ln f(x)) = \frac{f'(x)}{f(x)}$

\rightarrow Diff \times (1 over the function)

Use log rules first:

$$\ln(AB) = \ln A + \ln B$$

$$\ln(A/B) = \ln A - \ln B$$

$$\ln(A^n) = n \ln A$$

△ $\ln(x^n) = n \ln x$ (power comes down) is NOT $(\ln x)^n$

Note: $\ln e = 1$; $\sin 3x = (\sin x)^3 \rightarrow$ chain

Implicit: $\frac{d}{dx}(y^2) = 2y \cdot \left(\frac{dy}{dx}\right)$

△ $\frac{d^2x}{dy^2} \neq \left(\frac{dy}{dx}\right)^2$

6 Inverse trig — from the tables

$$y = \sin^{-1}(x/a) \rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{a^2 - x^2}}$$

$$y = \tan^{-1}(x/a) \rightarrow \frac{dy}{dx} = \frac{a}{a^2 + x^2}$$

a is a constant — identify it, drop into the format.

Chain versions:

$$\sin^{-1}(g) \rightarrow g'(x) / \sqrt{1 - g(x)^2}$$

$$\tan^{-1}(g) \rightarrow g'(x) / (1 + g(x)^2)$$

7 Slope of a tangent

Tangent line needs: a point + a slope

$m = \frac{dy}{dx}$ at the point (x, y)

$$\text{Tangent equation: } y - y_1 = m(x - x_1)$$

Other slope formulae:

- $m = (y_2 - y_1) / (x_2 - x_1)$ • $m = \text{rise/run}$
- Line $ax+by+c=0 \rightarrow m = -a/b$
- $m = \tan A$ (angle with positive x -axis)

★ At point of contact: slope of curve = slope of tangent

Got x ? Sub back to original to get y

8 Inc/Dec & max/min points

Increasing: $\frac{dy}{dx} > 0$ Decreasing: $\frac{dy}{dx} < 0$

Stationary / turning: $\frac{dy}{dx} = 0$

Max: $\frac{dy}{dx} = 0$ AND $\frac{d^2y}{dx^2} < 0$

Min: $\frac{dy}{dx} = 0$ AND $\frac{d^2y}{dx^2} > 0$

Inflection: $\frac{d^2y}{dx^2} = 0$

Max/Min word problems — 5 steps:

1. What's max/min of? 2. Formula: $A = \dots$ 3. Equation linking variables 4. Get one variable in terms of other 5. Sub into step 2, diff, set = 0

△ UNITS are vital — always state them!

Cubics — number of real roots:

- Max above axis, min below \rightarrow 3 distinct real roots
- One turning point on the axis \rightarrow 3 real roots, two equal
- Both turning points same side \rightarrow 1 real root

9 Rates of change

Type 1: Distance / Speed / Acceleration

- $s = f(t)$ = distance · $v = ds/dt$ = speed
- $a = d^2s/dt^2$ = acceleration
- Initial / at rest: $t = 0$ · Max speed: $ds/dt = 0$

Type 2: Linked rates (chain across rates)

$$dV/dt = (dV/dr) \times (dr/dt)$$

$$dr/dt \rightarrow m/s \quad dA/dt \rightarrow m^2/s \quad dV/dt \rightarrow m^3/s$$

Area/volume/SA formulas (circle, sphere, cylinder, cube) from the tables.

★ *What do we have? What do we need? LINK is in the question.*

⚠ DON'T FORGET UNITS

10 Inverse functions — finding f^{-1}

- f^{-1} swaps x and y — give the y , find the x
- f^{-1} is image of f under axial symmetry in $y = x$
- Only a bijective function has an inverse

⚠ $f^{-1}(x)$ is NOT $1/f(x)$

To find it:

1. Write $y = f(x)$ 2. Make x the subject 3. Write $f^{-1}(x)$

Quadratics: restrict domain, complete the square, then solve.