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PROOF
Exam Cog

① PROOF BY INDUCTION — THE METHOD

The same four lines, every single time:

1. Prove for $n = 1$ (or the first value stated)
2. Assume true for $n = k$
3. Prove for $n = k + 1$
4. Conclude: true for base AND $n = k+1 \Rightarrow$ true for all n

★ *Learn the skeleton cold — it never changes.*

② DIVISIBILITY (÷ BY A FIXED NUMBER)

At $n = k+1$ you always split into:

(multiple of the divisor) + (the assumption)

★ *Split the base: $5 = 4+1$, $9 = 8+1$, then $a^{p+q} = a^p \cdot a^q$*

✓ *TIP: pull the divisor out of the new term — the leftover IS the assumption.*

Comes up: 5^n-1 (÷4), 7^n-2^n (÷5), 4^n-1 (÷3), $3^{2n}-1$ (÷8), 9^n+3 (÷4), $5^{2n}-3^{2n}$ (÷8), $n(n+1)(2n+1)$ (÷6).

③ INEQUALITIES

At $n = k+1$: split off the assumed inequality, then show the leftover piece must be true.

★ *e.g. $2^{k+1} = 2^k + 2^k > k+1$ (assume $2^k > k$, must $2^k > 1$)*

Fractions: flip to compare denominators —

a bigger denominator ⇒ a smaller fraction.

⚠ WATCH: state the starting value — $n = 1$, or $n \geq 4$, or $n > 1$ — it's given.

④ SEQUENCE & SERIES (SUM FORMULAE)

LHS = Series, RHS = Algebra

★ *At $n = k+1$, add the next term T_{k+1} onto the assumed sum, then show both sides meet.*

✓ *TIP: the term to add is T_{k+1}; factor $(k+1)$ out of the RHS and tidy.*

Comes up: $1+2+\ldots+n = (n/2)(n+1)$; $1+3+\ldots+(2n-1) = n^2$; $\Sigma r^2 = (n/6)(n+1)(2n+1)$.

⑤ PROOF BY CONTRADICTION

To prove a statement is true, prove the opposite is false.

★ *Classic: $\sqrt{2}$ is irrational.*

Assume $\sqrt{2} = a/b$ (a, b no common factor) ⇒ $2b^2 = a^2$ ⇒ a even ⇒ $a = 2k$ ⇒ $b^2 = 2k^2$ ⇒ b even.

⇒ a, b both even = common factor 2 = contradiction ⇒ $\sqrt{2}$ is irrational.

Facts: even number = 2n; irrational ⇒ cannot be written as a fraction.

⑥ DE MOIVRE'S THEOREM (BY INDUCTION)

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

★ *At $n = k+1$, multiply the assumption by $(\cos\theta + i\sin\theta)$, then use compound-angle.*

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

→ collapses to $\cos(k+1)\theta + i\sin(k+1)\theta$. Done.