INTEGRATION

Exam Cog — Need to Know

① THE BASIC RULE

$\int x^n\,dx = x^{n+1}/(n+1) + c$

★ *Increase the power by 1, divide by the new power*

⚠ **WATCH:** NEVER forget + c — losing 3 marks for one missing constant!

Useful: x^(3/2) = (√x)³ = x√x

② GET INTO STANDARD FORM — 5 TYPES

Standard forms:

• $x^a + x^b + x^c + \ldots$ (separate terms)

• Maths tables formula

5 techniques to try:

Type 1: Multiply out brackets

Type 2: Use indices (rewrite roots/fractions)

Type 3: Factorise

Type 4: Look up in tables

Type 5: Split (single term on bottom → separate)

③ TRIG INTEGRATION

Type 1: Use the maths tables

Type 2: Product → sum/difference

From tables: 2 cos A sin B = sin(A+B) − sin(A−B)

⚠ **WATCH:** If question has cos A sin B (no 2!) → put ½ in front

Same idea for: 2 sin A sin B, 2 cos A cos B

④ EXPONENTIAL & LOG

$\int e^x\,dx = e^x + c$

$\int (1/x)\,dx = \ln|x| + c$ (in tables)

Cancel rules:

• ln $e^x = x$

• e^(ln x) = x

Note: $\log_e x = \ln x$

★ *$\int f'(x)\,dx = f(x) + c$ — integration UNDOES differentiation*

⑤ DEFINITE INTEGRALS

$\int_a^b f'(x)\,dx = f(b) - f(a)$

a, b are the LIMITS of integration

Method:

1. Integrate
2. Sub in TOP value
3. Sub in BOTTOM value
4. Subtract: top − bottom

⚠ **WATCH:** CARE WITH SIGNS — always use brackets when subtracting

No + c needed for definite integrals (cancels out)

⑥ FINDING c (INITIAL CONDITIONS)

If given a point or condition:

1. Integrate normally with + c
2. Sub in the given values
3. Solve for c

Example: If f'(x) = 2x and f(1) = 5

→ $f(x) = x^2 + c$

→ $5 = 1 + c$ → $c = 4$

→ $f(x) = x^2 + 4$

⑦ AREA UNDER A CURVE

Area under y = f(x), between x = a and x = b:

Area = $\int_a^b f(x)\,dx = \int_a^b y\,dx$

Area between y-axis and x = f(y):

Area = $\int_a^b f(y)\,dy = \int_a^b x\,dy$

Area BETWEEN two curves (top − bottom):

Area = $\int_a^b [f(x) - g(x)]\,dx$

★ *DRAW A DIAGRAM first — find where curve cuts x-axis (let y = 0)*

Area between curves: find intersections via simultaneous eqns

⑧ TRAPEZOIDAL RULE

Approximate area when integration is hard:

Area ≈ $(h/2) \cdot [y_1 + y_n + 2(y_2 + y_3 + \ldots + y_{n-1})]$

★ *Half the gap × [first + last + twice the rest]*

• h = width of each strip

• y values = heights at each strip edge

✓ **TIP:** Useful for irregular curves where ∫ doesn't work neatly

⑨ AVERAGE VALUE

Average value of f(x) on [a, b]:

Avg = $[1/(b-a)] \cdot \int_a^b f(x)\,dx$

★ *Total area ÷ width = average height*

⑩ DISTANCE, SPEED, ACCELERATION

Differentiate to go DOWN:

Distance → Speed → Acceleration

Integrate to go UP:

Acceleration → Speed → Distance

Symbols:

• s = f(t) = distance / displacement

• v = ds/dt = speed / velocity

• a = dv/dt = d^2s/dt^2 = acceleration

Conditions:

• Initial = at start = t = 0

• At rest / max distance: v = 0