Revise.ie

PROBABILITY

Exam Cog — Need to Know

① PERMUTATIONS — ARRANGE IN ORDER

Permutation = arrangement; ORDER MATTERS

★ AND = MULTIPLY   OR = ADD

n! = n(n−1)(n−2)…(3)(2)(1)

Factorial method: BREAK THE BIGGER ONE DOWN

n objects, take r: $^nP_r = n! / (n-r)!$

Together: use 'bubbles' (group together as one)

Not together: Total − Together

Identical objects: n! / (p! q!)

② COMBINATIONS — CHOOSE (no order)

^nC_r = (n choose r) = n! / [r!(n−r)!]

Type 2 — sub-divide: (m choose r) × (n choose s)

"Exactly" = one case to handle

"At least / at most" = multiple cases, ADD them

Points on diagram, no 3 collinear:

→ no 3 in a straight line

⚠️ **WATCH:** Permutation ≠ Combination — does ORDER matter?

③ BASIC PROBABILITY

P(E) = (favourable) / (total possible)

0 ≤ P(E) ≤ 1

- P(E) = 0 → impossible
- P(E) = 1 → certain

P(NOT E) = 1 − P(E)

Sample space: set of all possible outcomes

5 methods: Logic / Sample space / Tree diagram / Permutation / Combination

④ KEY EVENT TYPES

Relative frequency (experimental):

P(E) = times E happened / total trials

Exhaustive: P(E) + P(F) = 1 (no other outcomes)

Mutually Exclusive: can't happen together

→ P(A ∩ B) = 0

Independent: outcome of one doesn't affect the other

→ P(A ∩ B) = P(A) × P(B)

→ P(A|B) = P(A)

⑤ VENN DIAGRAMS

P(A ∪ B) = P(A) + P(B) − P(A ∩ B)

P(A') = everything outside A (complement)

P(A | B) = probability of A given B occurred

P(A | B) = P(A ∩ B) / P(B)

★ P(A ∩ B) = P(A) × P(B) ONLY if independent

To prove independent: check the multiplication rule holds

⑥ EXPECTED VALUE

Mean = μ = E(x) = Σ(x · P(x))

- x = the win / value
- P(x) = probability of that win
- Σ = sum of (sigma)

Method:

1. List each outcome and its probability
2. Multiply x by P(x)
3. Sum them all up

✓ **TIP:** Game is FAIR if E(x) = 0   Game in your favour if E(x) > 0

⑦ BERNOULLI TRIALS (BINOMIAL)

4 conditions:

1. Only 2 outcomes (success / failure)
2. Events independent
3. P(success) same each trial
4. Fixed number of trials

Formula (in log tables):

$P(r) = {}^nC_r \cdot p^r \cdot q^{n-r}$

p=success q=1−p n=trials r=successes

⚠️ **WATCH:** ONLY 2 outcomes allowed — no draws!

⑧ TREE DIAGRAMS

When to use: multi-stage events, with/without replacement

Method:

1. Draw branches for each stage
2. Write P(outcome) on each branch
3. MULTIPLY along branches (AND)
4. ADD between branches (OR)

★ Branches at each junction must sum to 1

⑨ COMMON QUESTION TYPES

With replacement: probabilities stay same each draw

Without replacement: denominator shrinks each draw

"At least one" trick:

P(at least 1) = 1 − P(none)

✓ **TIP:** Often easier to find the OPPOSITE event and subtract from 1

⑩ PROBABILITY SCALE & TIPS

0 —— 0.5 —— 1

impossible → even → certain

Quick reminders:

✓ Sense-check: 0 ≤ answer ≤ 1

✓ Independent before multiplying

✓ Conditional: P(A∩B) / P(B)

✓ Dice (1d6): each face = 1/6

✓ 52-card deck: 4 suits × 13 cards