SEQUENCES & SERIES

Exam Cog

① SEQUENCES vs SERIES

Sequence: a set of numbers/expressions following a rule
Series: the SUM of the terms of a sequence
Term: one element. T_n or U_n = general term
Format: $T_1, T_2, T_3, \ldots, T_{n-1}, T_n, T_{n+1}, \ldots$
$\mathbf{T_{n+1}}$ = the NEXT term up from T_n
Note: $n \in \mathbb{N}$ (whole positive)

② S_n AND THE BIG TRICK

$\mathbf{S_1 = U_1}$
$\mathbf{S_2 = U_1 + U_2}$
$\mathbf{S_3 = U_1 + U_2 + U_3}$
$\mathbf{S_n = U_1 + U_2 + \ldots + U_n} = \Sigma U_r$ (r=1 to n)

★ *Given S_n, find U_n:* $\boldsymbol{U_n = S_n - S_{n-1}}$

③ ARITHMETIC SEQUENCES (AP)

Rule: $U_n - U_{n-1}$ = constant (the common difference)
Notation: $a = U_1$ (first term) $d = U_2 - U_1$
General term: $\mathbf{T_n = a + (n - 1)d}$
Sum of first n terms:
$\mathbf{S_n = (n/2)[2a + (n - 1)d]}$

★ *AP series can NEVER have a limit — always divergent*

④ AP — QUESTION TYPES

Two terms given: simultaneous equations in a, d
3 expressions are first 3 terms?
e.g. x–3, 2x+7, x+5 → use $T_3 - T_2 = T_2 - T_1$
Sum/multiply to know values:
Find 3 terms summing to 12, multiplying to 48
→ let terms be **x – y, x, x + y**

✓ ***TIP:** Symmetric form makes the algebra tidy!*

⑤ GEOMETRIC SEQUENCES (GP)

Rule: T_{n+1} / T_n = constant (the common ratio)
Notation: a = first term $r = T_2 / T_1$
General term: $\mathbf{T_n = a \cdot r^{n-1}}$
Sum of first n terms:
$S_n = a(1 - r^n) / (1 - r)$, where $|r| \neq 1$

WATCH: Same letter a — but DIFFERENT meaning to AP. Don't confuse!

⑥ GP — QUESTION TYPES

Two terms given: simultaneous equations in a, r
3 expressions are first 3 terms?
e.g. 2, x+1, 32 → use $T_2/T_1 = T_3/T_2$
Sum/multiply to know values:
Find 3 terms summing to 14, multiplying to 64
→ let terms be **x/y, x, xy**

★ *AP uses x–y, x, x+y GP uses x/y, x, xy*

⑦ SUM TO INFINITY S_∞

ONLY for geometric series, only when $|r| < 1$:
$\mathbf{S_\infty = a / (1 - r)}$
$|r| < 1$ means r is a proper fraction
$|r| < 1$ → series **CONVERGES**
$|r| \geq 1$ → series **DIVERGES**

WATCH: Arithmetic series NEVER has S_∞ — it always diverges

⑧ RECURRENCE & INC / DEC

Recurrence relation: U_{n+1} depends on U_n
e.g. $U_{n+1} = 3U_n + 2$, with $U_1 = 1$
Increasing: $U_{n+1} > U_n$ for all n
equivalently $U_{n+1} / U_n > 1$
Decreasing: $U_{n+1} < U_n$ for all n
equivalently $U_{n+1} / U_n < 1$

⑨ INFINITE SERIES — GENERAL

$\mathbf{S_\infty = \lim_{(n \to \infty)} S_n}$
Step 1: find a tidy expression for S_n
Step 2: evaluate the limit as $n \to \infty$

★ ***Key limit:*** $\lim_{(n \to \infty)} r^n = 0$ *when* $|r| < 1$

Use rules from differentiation/limits to evaluate.

⑩ QUADRATIC SEQUENCE

If 'gap of the gap' is constant:
$\mathbf{T_n = an^2 + bn + c}$

★ *a = (gap of gap) / 2*

Method:
1. Find gap of gap → divide by 2 → get a
2. Sub n=1 and n=2 (with known T_1, T_2)
3. Solve simultaneously for b and c