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TRIGONOMETRY

Exam Cog

① RIGHT-ANGLED TRIANGLE — SOH CAH TOA

Hypotenuse: opposite the right-angle (longest)

Opposite: furthest from the angle

Adjacent: beside the angle

sin = O/H cos = A/H tan = O/A

★ *Silly Old Harry, Caught A Herring, Trawling Off America*

Pythagoras: $a^2 + b^2 = c^2$

② DEGREES, RADIANS, UNIT CIRCLE

360° in a circle 2π radians in a circle

π = 180° (in tables: 1 rad = 53°)

Convert: × π/180 (deg → rad) × 180/π (rad → deg)

Unit Circle: any point = (cos A, sin A)

★ *All Silly Tom Cats — quadrant signs*

• Q1: All + • Q2: Sin + • Q3: Tan + • Q4: Cos +

Reference angle rule (A in Q1):

Q2: 180−A Q3: 180+A Q4: 360−A

③ SIMPLE TRIG EQUATIONS

Method:

1. Identify quadrants from the SIGN
2. Find reference angle (Q1) — ignore sign
3. Apply quadrant rule for actual answers

If 2A in question: solve for 2A first, then ÷ 2

Add 360° to 2A repeatedly until A out of range

General form: A = α + 360°n

⚠ **WATCH: $-1 \le \sin A \le 1$; $-1 \le \cos A \le 1$; tan A unbounded**

④ COMPOUND ANGLES (A ± B)

All formulae in tables. Uses:

Type 1: Proofs (must know)

Type 2: Split angle: e.g. 75 = 30 + 45

Type 3: Given sin A & cos B → find sin/cos/tan of A+B

Type 4: Given tan(A+B) and tan A → find tan B

✓ **TIP:** 3 terms? Group 2 together: cos(45 − (A+B))

⑤ DOUBLE & HALF ANGLES

Double angle (in tables):

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = 2 \tan A / (1 - \tan^2 A)$

Half-angle: $\cos A = \cos^2(A/2) - \sin^2(A/2)$

★ *Halve angles on left → halve angles on right*

If you have ONE of sin A, cos A, tan A:

→ you can find the other two

⑥ PRODUCT ↔ SUM FORMULAE

In tables — common in integration questions

$2 \cos A \sin B = \sin(A+B) - \sin(A-B)$

$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$

$2 \cos A \cos B = \cos(A-B) + \cos(A+B)$

$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$

⚠ **WATCH: No '2' on the left? Multiply RHS by ½**

⑦ SOLVING TRIANGLES — DECISION TREE

Step 1: Read the question — draw it!

Step 2: Fill in all known information

Step 3: Ask: IS IT RIGHT-ANGLED?

YES → SOH CAH TOA or Pythagoras

NO → Sin Rule, Cosine Rule, or Area formula

Sin Rule: need ≥ 1 angle + opposite side

Cosine Rule: (a) 2 sides + included angle → 3rd side

(b) 3 sides → angle

Area: ½ ab sin C

★ *No length in triangle = useless triangle*

⑧ TRIG IDENTITIES

THE key identity:

$\sin^2 A + \cos^2 A = 1$

→ $\sin^2 A = 1 - \cos^2 A$

→ $\cos^2 A = 1 - \sin^2 A$

Method:

• Work each side SEPARATELY

• Write everything in sin and cos

• Only change a term if you GAIN by it

⚠ **WATCH: Small letters in identity? → use sin rule or cosine rule**

⑨ INVERSE TRIG & GRAPHS

$\sin^{-1}x = A$ means sin A = x

⚠ **WATCH: $\sin^{-1}x \neq 1/(\sin x)$ — they're different!**

y = a + b sin(cx) or **y = a + b cos(cx):**

• Period = 2π / c

• Range = [a − b, a + b]

Practise basic graphs of sin, cos, tan

⑩ KEY VOCAB & RULES

Horizontal: flat ground **Vertical:** standing

Elevation: UP from ground

Depression: DOWN from height

Direction: N, S, E, W

★ *LARGEST angle is OPPOSITE the LARGEST side*

3 question types: 2D, circle, 3D

⚠ **WATCH: UNITS are vital — always state them!**